

A Multivariate GARCH Model with Time-Varying Correlations

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First Draft: December 1998

Revised: November 2000

This Version: May 2001

Abstract: In this paper we propose a new multivariate GARCH model with time-varying correlations. We adopt the vech representation based on the conditional variances and the conditional correlations. While each conditional-variance term is assumed to follow a univariate GARCH formulation, the conditional-correlation matrix is postulated to follow an autoregressive moving average type of analogue. By imposing some suitable restrictions on the conditional-correlation-matrix equation, we construct a MGARCH model in which the conditional-correlation matrix is guaranteed to be positive definite during the optimization. Thus, our new model retains the intuition and interpretation of the univariate GARCH model and yet satisfies the positive-definite condition as found in the constant-correlation and BEKK models. We report some Monte Carlo results on the finite-sample distributions of the MLE of the varying-correlation MGARCH model. The new model is applied to some real data sets. It is found that extending the constant-correlation model to allow for time-varying correlations provides some interesting time histories that are not available in a constant-correlation model.

Key Words: BEKK model, constant correlation, Monte Carlo method, multivariate GARCH model, maximum likelihood estimate, varying correlation

JEL Classification: C12

Acknowledgement: Y.K. Tse acknowledges the support by the National University of Singapore Academic Research Grant RP-3981003. Albert Tsui is grateful to Ronald Lee for providing research support when he was on leave at the University of California at Berkeley. An earlier version of this paper was presented at the World Congress of the Econometric Society, Seattle, July 2000. We are indebted to the participants of the conference for their comments. We would also like to thank Jeffrey Wooldridge (the Editor), the Associate Editor and two anonymous referees for their many helpful suggestions. Any remaining errors are, of course, ours only. The email addresses of the authors are, respectively, ecstseyk@nus.edu.sg and ecsatsui@nus.edu.sg.

1 Introduction

Following the success of the autoregressive conditional heteroscedasticity (ARCH) model and the generalized ARCH (GARCH) model in describing the time-varying variances of economic data in the univariate case many researchers have extended these models to multivariate dimension. Applications of the multivariate GARCH (MGARCH) models to financial data have been numerous. For example, Bollerslev (1990) studied the changing variance structure of the exchange rate regime in the European Monetary System assuming the correlations to be time invariant. Kroner and Claessens (1991) applied the models to calculate the optimal debt portfolio in multiple currencies. Lien and Luo (1994) evaluated the multiperiod hedge ratios of currency futures in a MGARCH framework. Karolyi (1995) examined the international transmission of stock returns and volatility using different versions of MGARCH models. Baillie and Myers (1991) estimated the optimal hedge ratios of commodity futures and argued that these ratios are nonstationary. Gouriéroux (1997, Chapter 6) presented a survey of several versions of MGARCH models. See also Bollerslev, Chou and Kroner (1992) and Bera and Higgins (1993) for surveys on the methodology and applications of GARCH and MGARCH models.

Bollerslev, Engle and Wooldridge (1988) provided the basic framework for a MGARCH model. They extended the GARCH representation in the univariate case to the vectorized conditional-variance matrix. Their specification follows the traditional autoregressive moving average time series analogue. While this vech representation is very general, it involves a large number of parameters. Empirical applications require further restrictions and simplifications. A useful member of the vech-representation family is the diagonal form. Under the diagonal form, each variance-covariance term is postulated to follow a GARCH-type equation with the lagged variance-covariance term and the product of the corresponding lagged residuals as the right-hand-side variables in the

conditional-(co)variance equation.

It is often difficult to verify the condition that the conditional-variance matrix of an estimated MGARCH model is positive definite.¹ Furthermore, such conditions are often very difficult to impose during the optimization of the log-likelihood function. Bollerslev (1990) suggested the constant-correlation MGARCH (CC-MGARCH) model that can overcome these difficulties. He pointed out that under the assumption of constant correlations, the maximum likelihood estimate (MLE) of the correlation matrix is equal to the sample correlation matrix. As the sample correlation matrix is always positive definite, the optimization will not fail as long as the conditional variances are positive. In addition, when the correlation matrix is concentrated out of the log-likelihood function further simplification is achieved in the optimization..

Due to its computational simplicity, the CC-MGARCH model is widely used in empirical research. However, while the constant-correlation assumption provides a convenient MGARCH model for estimation, some studies find that this assumption is not supported by some financial data.² Thus, there is a need to extend the MGARCH models to incorporate time-varying correlations and yet retain the appealing feature of satisfying the positive-definite condition during the optimization..

Engle and Kroner (1995) proposed a class of MGARCH model called the BEKK (named after Baba, Engle, Kraft and Kroner) model. The motivation is to ensure the condition of a positive definite conditional-variance matrix in the process of optimization. Engle and Kroner provided some theoretical analysis of the BEKK model and related it to the vech-representation form. Another approach examines the conditional variance as a factor model. The works by Diebold and Nerlove (1989), Engel and Rodrigues (1989) and Engle, Ng and Rothschild (1990) are along this line. One disadvantage of

¹Engle, Granger and Kraft (1984) presented the necessary conditions for the conditional-variance matrix to be positive definite in a bivariate ARCH model. Extensions of these results to more general models are, however, intractable.

²For example, Tse (2000) found that the stock returns across different national markets exhibit time-varying correlations.

the BEKK and factor models is that the parameters cannot be easily interpreted, and their net effects on the future variances and covariances are not readily seen. Bera, Garcia and Roh (1997) reported that the BEKK model does not perform well in the estimation of the optimal hedge ratios. Lien, Tse and Tsui (2001) reported difficulties in getting convergence when using the BEKK model to estimate the conditional-variance structure of spot and futures prices.

In this paper we propose a new MGARCH model with time-varying correlations. Basically we adopt the vech representation. The variables of interest are, however, the conditional variances and conditional correlations. We assume a vech-diagonal structure in which each conditional-variance term follows a univariate GARCH formulation. The remaining task is to specify the conditional-correlation structure. We apply an autoregressive moving average type of analogue to the conditional-correlation matrix. By imposing some suitable restrictions on the conditional-correlation-matrix equation, we construct a MGARCH model in which the conditional-correlation matrix is guaranteed to be positive definite during the optimization. Thus, our new model retains the intuition and interpretation of the univariate GARCH model and yet satisfies the positive-definite condition as found in the constant-correlation and BEKK models.

The plan of the rest of the paper is as follows. In Section 2 we describe the construction of the varying-correlation MGARCH model. As in other MGARCH models, the new model can be estimated using the maximum likelihood estimation (MLE) method. Some Monte Carlo results on the finite-sample distributions of the MLE of the varying-correlation MGARCH model are reported in Section 3. Section 4 describes some illustrative examples of the new model using some real data sets. These are the exchange rate data, national stock market price data and sectoral stock price data. The new model is compared against the CC-MGARCH model and the BEKK model. It is found that the new model compares favourably against the BEKK model. Extending the constant-correlation model to allow for time-varying correlations provides some interest-

ing empirical results. The estimated conditional-correlation path provides a time history that would be lost in a constant-correlation model. Finally, we give some concluding remarks in Section 5.

2 A Varying-Correlation MGARCH Model

Consider a multivariate time series of observations $\{y_t\}$, $t = 1, \dots, T$, with K elements each, so that $y_t = (y_{1t}, \dots, y_{Kt})'$. We assume that the observations are of zero (or known) mean. This assumption simplifies the discussions without straining the notations.³

The conditional variance of y_t is assumed to follow the time-varying structure given by

$$\text{Var}(y_t | \Phi_{t-1}) = \Omega_t, \quad (1)$$

where Φ_t is the information set at time t . We denote the variance elements of Ω_t by σ_{it}^2 , for $i = 1, \dots, K$, and the covariance elements by σ_{ijt} , where $1 \leq i < j \leq K$. Denoting D_t as the $K \times K$ diagonal matrix with the i th diagonal element being σ_{it} , we let $\epsilon_t = D_t^{-1} y_t$. Thus, ϵ_t is the standardized residual and is assumed to be serially independently distributed with mean zero and variance matrix $\Gamma_t = \{\rho_{ijt}\}$. Of course, Γ_t is also the correlation matrix of y_t . Furthermore, $\Omega_t = D_t \Gamma_t D_t$.

To specify the conditional variance of y_t , we adopt the vech-diagonal formulation initiated by Bollerslev, Engle and Wooldridge (1988). Thus, each conditional-variance term follows a univariate GARCH(p, q) model given by the following equation

$$\sigma_{it}^2 = \omega_i + \sum_{h=1}^p \alpha_{ih} \sigma_{i,t-h}^2 + \sum_{h=1}^q \beta_{ih} y_{i,t-h}^2, \quad i = 1, \dots, K, \quad (2)$$

where ω_i , α_{ih} and β_{ih} are nonnegative, and $\sum_{h=1}^p \alpha_{ih} + \sum_{h=1}^q \beta_{ih} < 1$, for $i = 1, \dots, K$. Note that we may allow (p, q) to vary with i so that (p, q) should be regarded as the generic

³Additional parameters would be required to represent the conditional-mean equation in the complete model if the mean is unknown. Under certain conditions, the MLE of the parameters in the conditional-mean equation is asymptotically uncorrelated with the MLE of the parameters of the conditional-variance equation. Under such circumstances, we may treat y_t as pre-filtered observations (see Bera and Higgins (1993) for further discussions). Otherwise, the parameter vector has to be augmented to take account of the parameters in the unknown conditional mean.

order of the univariate GARCH process. Researchers adopting the vech-diagonal form typically assume that the above equation also applies to the conditional-covariance terms in which σ_{it}^2 is replaced by σ_{ijt} and y_{it}^2 is replaced by $y_{it} y_{jt}$ for $1 \leq i < j \leq K$. We shall, however, deviate from this approach. Specifically, we shall focus on the conditional-correlation matrix and adopt an autoregressive moving average analogue on this matrix. Thus, we assume that the time-varying conditional-correlation matrix Γ_t is generated from the following recursion

$$\Gamma_t = (1 - \theta_1 - \theta_2) \Gamma + \theta_1 \Gamma_{t-1} + \theta_2 \Psi_{t-1}, \quad (3)$$

where $\Gamma = \{\rho_{ij}\}$ is a (time-invariant) $K \times K$ positive definite parameter matrix with unit diagonal elements and Ψ_{t-1} is a $K \times K$ matrix whose elements are functions of the lagged observations of y_t .⁴ The functional form of Ψ_{t-1} will be specified below. The parameters θ_1 and θ_2 are assumed to be nonnegative with the additional constraint that $\theta_1 + \theta_2 \leq 1$. Thus, Γ_t is a weighted average of Γ , Γ_{t-1} and Ψ_{t-1} . Hence, if Ψ_{t-1} is a well-defined correlation matrix (i.e., positive definite with unit diagonal elements), Γ_t will also be a well-defined correlation matrix.⁵

It can be observed that Ψ_{t-1} is analogous to $y_{i,t-1}^2$ in the univariate GARCH(1, 1) model. However, as Γ_t is a standardized measure, we also require Ψ_{t-1} to depend on the (lagged) standardized residuals ϵ_t . Denoting $\Psi_t = \{\psi_{ijt}\}$, we propose to consider the following specification for Ψ_{t-1}

$$\psi_{ij,t-1} = \frac{\sum_{h=1}^M \epsilon_{i,t-h} \epsilon_{j,t-h}}{\sqrt{(\sum_{h=1}^M \epsilon_{i,t-h}^2)(\sum_{h=1}^M \epsilon_{j,t-h}^2)}}, \quad 1 \leq i < j \leq K. \quad (4)$$

Thus, Ψ_{t-1} is the sample correlation matrix of $\{\epsilon_{t-1}, \dots, \epsilon_{t-M}\}$. We define E_{t-1} as the $K \times M$ matrix given by $E_{t-1} = (\epsilon_{t-1}, \dots, \epsilon_{t-M})$. If B_{t-1} is the $K \times K$ diagonal matrix

⁴For the sake of simplicity and at the risk of being not thorough, we shall describe a correlation matrix as being positive definite. It is not difficult to see that for some statements made in this section, the term ‘‘positive definite’’ should, strictly speaking, be replaced by the term ‘‘positive semi-definite’’.

⁵This statement is subject to the condition that the recursion starts with a well-defined correlation matrix Γ_0 . Under such conditions, the diagonal elements of Γ_t are unity and Γ_t remains positive definite.

with the i th diagonal element being $(\sum_{h=1}^M \epsilon_{i,t-h}^2)^{1/2}$ for $i = 1, \dots, K$, then we have

$$\Psi_{t-1} = B_{t-1}^{-1} E_{t-1} E'_{t-1} B_{t-1}^{-1}. \quad (5)$$

Note that when $M = 1$, Ψ_{t-1} is identically equal to the matrix of unity. Updating the conditional-correlation matrix with respect to the matrix of unity is of course not meaningful. Thus, taking first-order lag for the formulation of Ψ_{t-1} is not sufficient. Indeed, $M \geq K$ is a necessary condition for Ψ_{t-1} to be positive definite. When positive-definiteness is satisfied, Ψ_{t-1} is a well-defined correlation matrix. Thus, the condition $M \geq K$ will be imposed subsequently.

Equation (3) is analogous to the univariate GARCH equation, with the additional restriction that the sum of the coefficients is equal to 1. Indeed, Γ_t involves updating the conditional-correlation matrix with respect to the latest conditional-correlation matrix Γ_{t-1} and a sample estimate of the conditional-correlation matrix based on the recent M standardized residuals. We shall call the model specified by (2), (3) and (5) the varying-correlation MGARCH (VC-MGARCH) model.

Assuming normality, $y_t | \Phi_{t-1} \sim N(0, D_t \Gamma_t D_t)$, so that (ignoring the constant term) the conditional log-likelihood ℓ_t of the observation y_t is given by

$$\ell_t = -\frac{1}{2} \ln |D_t \Gamma_t D_t| - \frac{1}{2} y_t' D_t^{-1} \Gamma_t^{-1} D_t^{-1} y_t \quad (6)$$

$$= -\frac{1}{2} \ln |\Gamma_t| - \frac{1}{2} \sum_{i=1}^K \ln \sigma_{it}^2 - \frac{1}{2} y_t' D_t^{-1} \Gamma_t^{-1} D_t^{-1} y_t, \quad (7)$$

from which we can obtain the log-likelihood function of the sample as $\ell = \sum_{t=1}^T \ell_t$. Here the log-likelihood function is conditional on Γ_0 , Ψ_0 and y_0 being fixed. These assumptions have no effects on the asymptotic distribution of the MLE. Denoting $\theta = (\omega_1, \alpha_{11}, \dots, \alpha_{1p}, \beta_{11}, \dots, \beta_{1q}, \omega_2, \dots, \beta_{Kq}, \rho_{12}, \dots, \rho_{K-1,K}, \theta_1, \theta_2)$ as the parameter vector of the model, the MLE of θ is obtained by maximizing ℓ with respect to θ . We shall denote this value by $\hat{\theta}$.

For parameter parsimony, (p, q) is usually taken to be of low order. For $p = q = 1$,

the total number of parameters in the VC-MGARCH model is $3K + K(K + 1)/2 + 2$. In comparison, an unrestricted BEKK(1, 1) model has $K(K + 1)/2 + 2K^2$ parameters. For example, for $K = 2, 3$ and 4 , the number of parameters in the VC-MGARCH model is $9, 14$ and 20 , respectively, while that for the BEKK model is $11, 24$ and 42 , respectively. The number of parameters in the VC-MGARCH model always exceeds that of the constant-correlation model by 2 , due to the parameters θ_1 and θ_2 . Indeed the CC-MGARCH model is nested within the VC-MGARCH model under the restrictions $\theta_1 = \theta_2 = 0$.

The conditions $0 \leq \theta_1, \theta_2 \leq 1$ and $\theta_1 + \theta_2 \leq 1$ pose some problems in the optimization. One way to get around this difficulty is through transformation. For example, we may define $\theta_i = \lambda_i^2 / (1 + \lambda_1^2 + \lambda_2^2)$ for $i = 1, 2$, where λ_1 and λ_2 are unrestricted parameters. The log-likelihood function may be initially optimized with respect to λ_1, λ_2 and other parameters of interest. The optimization is then shifted to the original vector θ when convergence with respect to λ_1, λ_2 and other parameters has been achieved. This technique is used in the computations reported in this paper.

3 Some Monte Carlo Results

Research on the asymptotic theory of conditional heteroscedasticity models has been lagging behind their empirical applications. Weiss (1986), Pantula (1989), Bollerslev and Wooldridge (1992), Lee and Hansen (1994), Lumsdaine (1996) and Ling and Li (1997b) investigated the asymptotic distribution of the quasi MLE (QMLE) of the univariate ARCH/GARCH models. Sufficient conditions for consistency and asymptotic normality have been established. Recently, Ling and McAleer (2000) examined the asymptotic distribution of a class of vector ARMA-GARCH models. They established conditions for strict stationarity and ergodicity, and proved the consistency and asymptotic normality of the QMLE under some mild moment conditions. While the models considered by Ling

and McAleer are quite general, the CC-GARCH framework is adopted and time-varying conditional correlation is not allowed. An extension of the results by Ling and McAleer to the VC-MGARCH model will be interesting. This, however, is beyond the scope of this paper.

An interesting issue for empirical applications concerns the properties of the MLE of the conditional heteroscedasticity models in small and moderate samples. In the univariate case, Engle, Hendry and Trumble (1985) and Lumsdaine (1995) examined the small-sample properties of the MLE of the ARCH and GARCH models. In this section we report some results on the small-sample properties of the MLE of the VC-MGARCH model based on a small-scale Monte Carlo experiment. It is not our intention to provide a comprehensive Monte Carlo study of the MLE. We shall focus our interest on the small-sample bias and mean squared error only. The reliability of the inference concerning the model parameters will not be examined. Our results, however, will provide some preliminary evidence with respect to the small-sample properties of the MLE of the VC-MGARCH model.

We consider bivariate VC-MGARCH models in which the conditional-variance equations are given by

$$\sigma_{it}^2 = \omega_i + \alpha_i \sigma_{i,t-1}^2 + \beta_i y_{i,t-1}^2, \quad i = 1, 2, \quad (8)$$

with

$$\rho_t = (1 - \theta_1 - \theta_2) \rho + \theta_1 \rho_{t-1} + \theta_2 \psi_{t-1}, \quad (9)$$

where ψ_{t-1} is specified as

$$\psi_{t-1} = \frac{\sum_{h=1}^2 \epsilon_{1,t-h} \epsilon_{2,t-h}}{\sqrt{(\sum_{h=1}^2 \epsilon_{1,t-h}^2)(\sum_{h=1}^2 \epsilon_{2,t-h}^2)}}. \quad (10)$$

with $\epsilon_{it} = y_{it} / \sigma_{it}$ for $i = 1, 2$.⁶

We consider four experimental setups. The true parameter values of the data generating processes of these experiments, labelled E1 through E4, are given in Tables 1.1

⁶All computations reported in this paper assume $M = K$ in the definition of Ψ_t .

and 1.2. Observations $\{y_t\}$ are generated from these models assuming the errors are normally distributed. We consider $T = 500, 1000$ and 1500 . The MLE are calculated for each generated sample. Using Monte Carlo samples of 1000 runs, we estimate the bias and mean squared error (MSE) of the MLE.

E1 and E2 represent models with higher volatility persistence (as measured by $\alpha_i + \beta_i$), while E3 and E4 represent models with lower volatility persistence. The selected values of ρ in the experiments are 0.2 and 0.7. It can be seen from the Monte Carlo results that the biases of the MLE are generally quite small. The bias decreases with the sample size, although in some cases not steadily. Likewise, the same is true for the MSE. Overall, for the sample sizes and models considered, the bias and MSE appear to be small.

In the next section, we illustrate the application of the VC-MGARCH model with some real data sets.

4 Some Illustrative Examples

We examine three sets of financial data, denoted by DS1, DS2 and DS3. DS1 consists of two exchange rate (versus US dollar) series, namely, the Deutschemark (D) and the Japanese Yen (J). These series represent 2131 daily observations from January 1990 through June 1998. DS2 covers the stock market indices of the Hong Kong and the Singapore markets. We use the Hang Seng Index (H) for the Hong Kong market and the SES Index (S) for the Singapore market. There are 1942 daily (closing) prices for each series, covering the period from January 1990 through March 1998. DS3 consists of three sectoral price indices of the Hong Kong stock market. These are the Finance (F), Properties (P) and Utilities (U) sectors. Each series have 2440 daily observations covering the period from October 1990 through August 2000. DS1 was downloaded from the website of the Federal Reserve Bank of New York. DS2 was compiled from various

issues of the Stock Exchange of Singapore Journal. Some adjustments were made to account for the differences in the holidays of the two exchanges. DS3 was downloaded from Datastream.

Figures 1 through 3 present the plots of the seven series in the three data sets. In Figure 1 the Japanese Yen (Y) series have been rescaled for easy presentation. This is similarly done for the Hang Seng Index (H) series in Figure 2. We can see that the exchange rates of the Deutschemark and the Japanese Yen generally moved in tandem against the US dollar during the sample period. As expected, the three sectoral indices in the Hong Kong stock market moved quite closely together. This is especially true for the Finance and Properties Indices. In contrast, the Utilities Index was quite sluggish in the mid 90s while the Finance and Properties Indices were undergoing a bull run during this period. It is quite clear from Figure 2 that the national stock markets of Hong Kong and Singapore experienced different phases of bulls and bears. The general impression is that Hong Kong has a more volatile market compared to Singapore.

Table 2 provides a summary of the descriptive statistics of the data. The summary statistics refer to those of the differences of the logarithmic series (expressed in percentage). It can be seen that all differenced logarithmic series exhibit excess kurtosis (compared to the normal distribution) in the unconditional distribution. While the exchange rate data (DS1) demonstrate no evidence of serial correlation, the stock return data (DS2 and DS3) have significant serial correlation as suggested by the Q_1 statistics. The Q_2 statistics show that there is serial correlation in the conditional variance for all data sets and GARCH type of modelling may be required.⁷ In the subsequent analysis, we apply MGARCH models to the data sets. Autoregressive filters are used for the conditional-mean equations. Thus, the following conditional-mean equations are

⁷ $Q_1(20)$ is the Box-Pierce portmanteau statistic of the differenced logarithmic series based on the autocorrelation coefficients up to order 20. Similarly, $Q_2(20)$ is the portmanteau statistic of the squared differenced logarithmic series.

considered

$$y_{jt} = \mu_j + \sum_{i=1}^{p_j} \phi_{ji} y_{j,t-i} + \epsilon_{jt}, \quad j = 1, \dots, K, \quad (11)$$

where $\epsilon_t | \Phi_{t-1} \sim N(0, \Omega_t)$. The types of MGARCH model we consider are: the CC-MGARCH(1, 1) model, the VC-MGARCH(1, 1) model and the BEKK(1, 1) model. The parameters of the conditional-mean and conditional-variance equations are estimated simultaneously using MLE assuming normality.

We fit the CC-MGARCH(1, 1) model to the data sets using Bollerslev's (1990) algorithm. The results are summarized in Table 3.1.⁸ It can be seen that the estimates of α , β and ρ are statistically significant at the 5 percent level for all data sets. In comparison, the exchange rate data have the highest intensity of persistence in volatility as measured by $\hat{\alpha} + \hat{\beta}$. With respect to the correlation coefficients, the returns of the national stock markets of Hong Kong and Singapore have the lowest correlation. In contrast, the correlations between the various sectoral indices of the Hong Kong stock market are the highest.

Table 3.2 summarizes the estimation results of the VC-MGARCH(1, 1) models for the three data sets.⁹ Again, it can be seen that the estimates of α , β and ρ are statistically significant at the 5 percent level for all data sets. In addition, all estimates of θ_1 and θ_2 are statistically significant at the 5 percent level, indicating that the correlations are significantly time varying. We note that the intensity of the volatility persistence remains approximately unchanged compared to the CC-MGARCH models. Indeed, incorporating time-varying correlations does not have much effect on the estimates of α and β . The estimates of ρ in the varying-correlation models are all larger than the corresponding estimates of ρ in the constant-correlation models. This, however, does not imply that the correlations are on average higher in the varying-correlation model.

⁸All estimations in this paper are coded in GAUSS. The standard errors are calculated using the robustified quasi-MLE (QMLE) covariance matrix of the parameters.

⁹The results are based on the assumption $M = K$. We have re-estimated the models with $M = K + 1$. The results are qualitatively similar.

It should be noted that the time-invariant component of the conditional correlation coefficient in the VC-MGARCH(1, 1) model is $(1 - \theta_1 - \theta_2)\rho$. A comparison of the correlation correlations in the two models will be provided below.

We also estimate the BEKK(1, 1) model defined by the following conditional-variance equation¹⁰

$$\begin{aligned} \Omega_t = & \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix}' \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix} + \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}' \Omega_{t-1} \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \\ & + \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}' \epsilon_{t-1} \epsilon_{t-1}' \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}. \end{aligned} \quad (12)$$

Tables 3.3 and 3.4 summarize the estimation results.¹¹ It is found that all off diagonal elements of the GARCH (i.e., g_{ij}) and ARCH (i.e., a_{ij}) terms of the conditional-variance equations are insignificant. Thus, Table 3.4 presents the diagonal BEKK(1, 1) models.

As the CC-MGARCH(1, 1) model is nested within the VC-MGARCH(1, 1) model, ignoring the extension would induce model misspecification. We now proceed to examine the model diagnostics of the estimated models. Table 4 summarizes the maximized log-likelihood value (LF) and a battery of diagnostic tests for the fitted models. The constant-correlation assumption is tested using a Lagrange multiplier test (LMC) based on the estimates of the CC-MGARCH(1, 1) model and the likelihood ratio test (LR) based on the estimates of the VC-MGARCH(1, 1) model. LMC is the Lagrange multiplier test suggested by Tse (2000) for the assumption of (joint) constant correlation in a MGARCH model. It is asymptotically distributed as χ_R^2 , where $R = K(K - 1)/2$, under the null.¹² From Panel A of Table 4 we can see that the constant-correlation assumption is rejected for all data sets at the 5 percent level of significance. In Panel B of the table we present the likelihood ratio statistic LR , which tests for the restriction $H_0 : \theta_1 = \theta_2 = 0$. It can be seen that the constant-correlation assumption is rejected for

¹⁰This equation is for the bivariate case. The trivariate model is similarly defined.

¹¹The BEKK(1, 1) models are estimated using the GAUSS application module GAUSSX.

¹²Tse (2000) provided some Monte Carlo results for the finite-sample distributions of the LMC test. In particular, he showed that the test is robust against nonnormality in moderate samples.

all data sets at any conventional level of significance.

To further test for misspecification in the MGARCH models we adopt the regression-based diagnostics suggested by Wooldridge (1990, 1991). The methodology developed by Wooldridge applies to a wide class of possible misspecification. Here we focus on the problem of misspecification in the conditional heteroscedasticity. As shown by Wooldridge, the suggested tests are robust to departure from distributional assumptions that are not being tested. Since our main concern is misspecification in the conditional variance, we use the squared standardized residuals and the cross products of the squared standardized residuals as the indicators.

We first consider tests based on the squared standardized residuals. We denote $\hat{\epsilon}_{it}$ as the estimate of the standardized residual ϵ_{it} and $\hat{\sigma}_{it}^2$ as the estimated conditional variance of y_{it} . We define $\hat{\lambda}_{it} = (\hat{\epsilon}_{i,t-1}^2, \hat{\epsilon}_{i,t-2}^2, \dots, \hat{\epsilon}_{i,t-Q}^2)'$ as the vector of indicator variables, and $\nabla_{\theta} \hat{\sigma}_{it}^2$ as the gradient vector of σ_{it}^2 with respect to θ evaluated at $\hat{\theta}$. Denoting $(\nabla_{\theta} \hat{\sigma}_{it}^2) / \hat{\sigma}_{it}^2$ as $\nabla_{\theta} \tilde{\sigma}_{it}^2$, we regress each element of $\hat{\lambda}_{it}$ on $\nabla_{\theta} \tilde{\sigma}_{it}^2$ to obtain the Q -element residuals \hat{r}_{it} . Finally, we regress unity on the vector of Q regressors $\hat{\phi}_{it} \hat{r}_{it}$, where $\hat{\phi}_{it} = \hat{\epsilon}_{it}^2 - 1$. We calculate $W_{ii}(Q) = T - SSR$, where SSR is the sum of squares of the residuals of the last regression. If there is no model misspecification, $W_{ii}(Q)$ is asymptotically distributed as χ_Q^2 .

The above diagnostic statistic can be calculated for the cross products of the standardized residuals from different equations as tests for pairwise correlations. Specifically, we define $\hat{\lambda}_{ijt} = (\hat{\epsilon}_{i,t-1} \hat{\epsilon}_{j,t-1}, \hat{\epsilon}_{i,t-2} \hat{\epsilon}_{j,t-2}, \dots, \hat{\epsilon}_{i,t-Q} \hat{\epsilon}_{j,t-Q})'$ and $\nabla_{\theta} \tilde{\phi}_{ijt}$ as the gradient vector of $\phi_{ijt} = \epsilon_{it} \epsilon_{jt} - \rho_{ijt}$ with respect to θ evaluated at $\hat{\theta}$. We regress each element of $\hat{\lambda}_{ijt}$ on $\nabla_{\theta} \tilde{\phi}_{ijt}$ to obtain the Q -element residuals \hat{r}_{ijt} , and then regress unity on the Q regressors $\hat{\phi}_{ijt} \hat{r}_{ijt}$, where $\hat{\phi}_{ijt} = \hat{\epsilon}_{it} \hat{\epsilon}_{jt} - \hat{\rho}_{ijt}$. We define the test statistic as $W_{ij}(Q) = T - SSR$ for $1 \leq i < j \leq K$, which is asymptotically distributed as χ_Q^2 when there is no misspecification.¹³

¹³In this paper the gradient vectors required for the computation of the W statistics are calculated

We apply the W statistics to the MGARCH models with $Q = 4$. From the results in Table 4 we can see that both the CC-MGARCH and the VC-MGARCH models pass the diagnostic checks of the W statistics. Indeed, the W statistics of the two models are quite similar. As the constant-correlation assumption is not supported by the LMC and the LR statistics, one might expect the W statistics of the CC-MGARCH model to be significant. The fact that this is not the case may be an indication of loss in power when the test has no specific alternative. As for the BEKK model, most diagnostics are insignificant, with the exception of W_{11} in DS3. It is noted that the CC-MGARCH model has the lowest log-likelihood for all models. The VC-MGARCH model has the highest log-likelihood for DS2 and DS3, while the BEKK model has the highest log-likelihood for DS1. Based on penalized likelihood criterion such as the AIC, VC-MGARCH is the preferred model for DS2 and DS3, and BEKK is the preferred model for DS1.

In Table 5 we present the summary statistics of the standardized residuals of the fitted models. It can be seen that the standardized kurtosis and the Q_2 statistics have dropped significantly compared to those of the raw data in Table 2.¹⁴ While most Q statistics seem to be low, there is an exception for Q_2 in the F series of DS3.¹⁵

To obtain a clearer picture of the time history of the conditional correlations, we plot the time paths of the conditional correlations based on the VC-MGARCH and BEKK models. The plots are presented in Figures 4 through 8. It can be seen from the graphs that the conditional correlations estimated from the VC-MGARCH and BEKK models follow each other quite closely. However, the paths based on the BEKK model have much larger variability than those estimated by the VC-MGARCH model. In what follows we describe the conditional-correlation paths as provided by the VC-MGARCH model in

using numerical differentiation.

¹⁴We note that the Q_1 and Q_2 statistics are presented here for completeness. As pointed out by Li and Mak (1994) and Ling and Li (1997a) these statistics are not distributed as χ^2 under the null of no misspecification. While some of the Q_1 statistics appear to be large, we report that none of the lag autocorrelation coefficients is larger than 0.08 in absolute value.

¹⁵Note that W_{11} in Panel C of Table 4 is also found to be significant.

some detail.

Figure 4 presents the correlations between of the Deutschemark and the Japanese Yen. Largely, there are two subperiods when the conditional correlations of these two currencies were mostly above the average (constant) level, namely, October 1991 to June 1993 and March 1994 to October 1996. From October 1996 to June 1998, the conditional correlations were mostly below the average level.

Figure 5 presents an interesting case in which we can see that the conditional correlations between the Hong Kong and the Singapore stock markets were experiencing an upward shift. From 1994 onwards, the conditional correlations were mostly above the average level, whereas the reverse was true before 1994. This finding has important implications for the international diversification of equity portfolios. While the increasing conditional correlations means that the two national markets were becoming more closely integrated, it also implies that there is reducing benefits for international diversification. Using moving windows of unconditional correlations, Longin and Solnik (1995) showed that there was evidence of increasing correlations between international stock markets in 1960 – 1990.¹⁶ Our similar finding for the Hong Kong and the Singapore markets is commensurate with the increasing importance of intra-Asian business in the 90s.¹⁷

Figures 6 through 8 show that the pairwise correlations between the three sectors in the Hong Kong stock market are quite similar. Broadly speaking, the conditional correlations were above average in the subperiods of 1993 to 1994 and mid-1997 to mid-1999. These two subperiods coincide with the time when the Hong Kong stock market was experiencing a downturn. In contrast, during the subperiods of the bull runs from 1995 to mid-1997 and post mid-1999, the conditional correlations were below average.

¹⁶For an update of the correlations of international stock markets in the recent crisis period, see Longin and Solnik (2001).

¹⁷In the second half of the 90s, many companies with business activities in Hong Kong were listed on the Singapore exchange. The most notable example is the listing of the five companies in the Jardine group.

At the risk of over-simplification, this casual observation agrees with the hypothesis that contagion is stronger for negative returns than for positive returns.¹⁸ For the BEKK model, the conditional correlations are quite unstable in some periods.

We shall end this section by stating that it is not our intention to claim that the VC-MGARCH models as presented here represent the best MGARCH models for the data. Other MGARCH models could also provide the conditional-correlation structure. The VC-MGARCH model, however, does provide a viable alternative that is relatively easy to estimate. As the examples have illustrated, modelling correlations as a time-varying structure provides some interesting results that are not obtainable from constant-correlation models.

5 Conclusions

In this paper we propose a new MGARCH model with time-varying correlations. We assume a vech-diagonal structure in which each conditional-variance term follows a univariate GARCH formulation. The remaining task is to specify the conditional-correlation structure. We apply an autoregressive moving average type of analogue to the conditional-correlation matrix. By imposing some suitable restrictions on the conditional-correlation-matrix equation, we construct a MGARCH model in which the conditional-correlation matrix is guaranteed to be positive definite during the optimization.

We report some Monte Carlo results on the finite-sample distributions of the MLE of the varying-correlation MGARCH model. It is found that the bias and MSE of the MLE are small for sample sizes of 500 or above. The new model is applied to three data sets, namely, the exchange rate data, the national stock market data and the sectoral price data. The new model is found to pass the model diagnostics satisfactorily and

¹⁸Bae, Karolyi and Stulz (2000) examined the financial contagion among Asian and Latin American economies using a multinomial logit model. They reported that the evidence of contagion being stronger for negative returns than for positive returns is mixed.

compare favourably against the BEKK model, while the constant-correlation MGARCH model is found to be inadequate. Extending the constant-correlation model to allow for time-varying correlations provides some interesting empirical results. In particular, the estimated conditional-correlation path provides an interesting time history that would not be available in a constant-correlation model.

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Table 1.1: Estimated Bias and MSE of the MLE of Bivariate VC-MGARCH(1, 1) Models

Parameters	Experiment: E1				Experiment: E2			
	True Value	Sample Size	Bias	MSE	True Value	Sample Size	Bias	MSE
ω_1	0.4	500	0.0907	0.0687	0.4	500	0.1166	0.0993
		1000	0.0363	0.0194		1000	0.0487	0.0273
		1500	0.0266	0.0116		1500	0.0328	0.0157
α_1	0.8	500	-0.0135	0.0033	0.8	500	-0.0183	0.0043
		1000	-0.0056	0.0012		1000	-0.0070	0.0016
		1500	-0.0046	0.0008		1500	-0.0050	0.0010
β_1	0.15	500	-0.0007	0.0013	0.15	500	0.0005	0.0017
		1000	-0.0005	0.0006		1000	-0.0010	0.0008
		1500	0.0005	0.0004		1500	-0.0004	0.0005
ω_2	0.2	500	0.0313	0.0095	0.2	500	0.0364	0.0118
		1000	0.0132	0.0031		1000	0.0123	0.0040
		1500	0.0076	0.0017		1500	0.0089	0.0024
α_2	0.7	500	-0.0170	0.0062	0.7	500	-0.0230	0.0079
		1000	-0.0094	0.0023		1000	-0.0075	0.0031
		1500	-0.0043	0.0015		1500	-0.0047	0.0018
β_2	0.2	500	-0.0018	0.0023	0.2	500	0.0011	0.0030
		1000	0.0013	0.0010		1000	-0.0003	0.0013
		1500	-0.0005	0.0008		1500	-0.0005	0.0009
ρ	0.7	500	-0.0011	0.0028	0.2	500	-0.0008	0.0077
		1000	-0.0027	0.0084		1000	-0.0012	0.0034
		1500	0.0010	0.0009		1500	0.0001	0.0022
θ_1	0.8	500	-0.0018	0.0014	0.8	500	-0.0358	0.0181
		1000	-0.0090	0.0023		1000	-0.0194	0.0065
		1500	0.0011	0.0004		1500	-0.0111	0.0029
θ_2	0.1	500	-0.0006	0.0008	0.1	500	0.0043	0.0016
		1000	-0.0064	0.0014		1000	0.0023	0.0008
		1500	0.0005	0.0003		1500	0.0011	0.0004

Notes: See equations (8), (9) and (10) for the data generating processes.

Table 1.2: Estimated Bias and MSE of the MLE of Bivariate VC-MGARCH(1, 1) Models

Parameters	Experiment: E3				Experiment: E4			
	True Value	Sample Size	Bias	MSE	True Value	Sample Size	Bias	MSE
ω_1	0.4	500	0.0293	0.0148	0.4	500	0.0315	0.0188
		1000	0.0137	0.0062		1000	0.0114	0.0088
		1500	0.0090	0.0037		1500	0.0051	0.0052
α_1	0.5	500	-0.0131	0.0092	0.5	500	-0.0181	0.0109
		1000	-0.0077	0.0036		1000	-0.0067	0.0051
		1500	-0.0053	0.0022		1500	-0.0025	0.0031
β_1	0.3	500	-0.0050	0.0037	0.3	500	-0.0032	0.0042
		1000	-0.0007	0.0017		1000	-0.0019	0.0021
		1500	0.0003	0.0011		1500	-0.0022	0.0015
ω_2	0.2	500	0.0197	0.0067	0.2	500	0.0219	0.0081
		1000	0.0089	0.0026		1000	0.0109	0.0032
		1500	0.0054	0.0016		1500	0.0089	0.0024
α_2	0.5	500	-0.0291	0.0216	0.5	500	-0.0352	0.0268
		1000	-0.0125	0.0090		1000	-0.0188	0.0110
		1500	-0.0083	0.0054		1500	-0.0089	0.0074
β_2	0.2	500	-0.0028	0.0030	0.2	500	-0.0008	0.0034
		1000	-0.0017	0.0014		1000	0.0016	0.0017
		1500	0.0001	0.0009		1500	0.0021	0.0013
ρ	0.7	500	0.0011	0.0064	0.2	500	0.0002	0.0139
		1000	0.0014	0.0025		1000	0.0007	0.0068
		1500	-0.0003	0.0015		1500	0.0001	0.0041
θ_1	0.6	500	-0.0026	0.0034	0.6	500	-0.0137	0.0055
		1000	-0.0015	0.0014		1000	-0.0035	0.0023
		1500	-0.0011	0.0010		1500	-0.0058	0.0016
θ_2	0.3	500	-0.0048	0.0019	0.3	500	0.0035	0.0023
		1000	-0.0019	0.0009		1000	-0.0009	0.0010
		1500	-0.0006	0.0006		1500	0.0019	0.0007

Notes: See equations (8), (9) and (10) for the data generating processes.

Table 2: Summary Statistics of the Differenced Logarithmic Series of Various Data Sets

Variable (Code)	Mean	Std Dev	Minimum	Maximum	Std Skewness	Std Kurtosis	$Q_1(20)$	$Q_2(20)$	No of Obs
<u>Panel A: Forex Market Data (DS1), 90/1 - 98/6</u>									
Deutschmark (D)	0.0025	0.6746	-2.8963	3.1030	0.3715	16.6655	21.9957	464.2324	2131
Japanese Yen (J)	-0.0023	0.6750	-4.5228	3.2269	-9.5384	33.4012	27.6373	112.5759	2131
<u>Panel B: Stock Market Data (DS2), 90/1 - 98/3</u>									
Hong Kong (H)	0.0721	1.7093	-14.7347	17.2471	-0.0533	120.2852	36.6618	759.7676	1942
Singapore (S)	-0.0010	1.0768	-7.7236	8.7867	-1.6643	95.7543	116.8250	846.2872	1942
<u>Panel C: Hang Seng Sectoral Indices Data (DS3), 90/10 - 00/8</u>									
Finance (F)	0.1055	1.8196	-17.6894	18.0011	-2.7550	109.8381	57.6920	986.0076	2440
Properties (P)	0.0561	2.1585	-14.2739	20.6846	5.5556	88.3062	93.1470	986.3667	2440
Utilities (U)	0.0667	1.6940	-14.4889	16.6176	6.6282	92.1913	42.5415	609.1001	2440

Notes: $Q_1(20)$ is the Box-Pierce portmanteau statistic of the differenced logarithmic series based on the autocorrelation coefficients up to order 20. Similarly, $Q_2(20)$ is the portmanteau statistic of the squared differenced logarithmic series.

Table 3.1: Estimation Results of the Constant-Correlation Models: CC-MGARCH(1, 1)

Variable	μ	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ω	α	β	Correlations
<u>Panel A: Forex Market Data (DS1)</u>									
D	0.0004 (0.0128)	0.0470 (0.0208)				0.0070 (0.0034)	0.9353 (0.0151)	0.0485 (0.0097)	$\rho_{DJ} = 0.5241$ (0.0170)
J	-0.0055 (0.0136)	0.0474 (0.0220)				0.0103 (0.0074)	0.9308 (0.0320)	0.0484 (0.0192)	
<u>Panel B: National Stock Market Data (DS2)</u>									
H	0.1202 (0.0302)	0.0851 (0.0261)				0.2123 (0.0850)	0.7730 (0.0577)	0.1385 (0.0328)	$\rho_{HS} = 0.3129$ (0.0254)
S	0.0256 (0.0195)	0.1720 (0.0255)				0.0915 (0.0299)	0.7249 (0.0652)	0.1891 (0.0492)	
<u>Panel C: Hang Seng Sectoral Indices Data (DS3)</u>									
F	0.1043 (0.0288)	0.0843 (0.0173)	0.0471 (0.0155)			0.1683 (0.0615)	0.8324 (0.0441)	0.1078 (0.0280)	$\rho_{FP} = 0.7658$ (0.0114)
P	0.0560 (0.0319)	0.1437 (0.0169)	0.0324 (0.0146)	-0.0283 (0.0142)	-0.0226 (0.0109)	0.1304 (0.0374)	0.8731 (0.0231)	0.0904 (0.0157)	$\rho_{FU} = 0.6807$ (0.0148)
U	0.0305 (0.0269)	0.0834 (0.0181)		-0.0343 (0.0160)		0.2070 (0.0454)	0.7880 (0.0312)	0.1277 (0.0205)	$\rho_{PU} = 0.7138$ (0.0151)

Notes: The parameter estimates are the MLE assuming normality. The figures in parentheses are the standard errors. They are calculated using the robustified quasi-MLE (QMLE) covariance matrix of the parameters.

Table 3.2: Estimation Results of the Varying-Correlation Models: VC-MGARCH(1, 1)

Variable	μ	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ω	α	β	θ_1	θ_2	Correlations
<u>Panel A: Forex Market Data (DS1)</u>											
D	0.0013 (0.0136)	0.0513 (0.0198)				0.0055 (0.0030)	0.9376 (0.0141)	0.0504 (0.0098)	0.9705 (0.0081)	0.0158 (0.0046)	$\rho_{DJ} = 0.6292$ (0.0457)
J	-0.0011 (0.0155)	0.0494 (0.0219)				0.0103 (0.0070)	0.9314 (0.0303)	0.0473 (0.0180)			
<u>Panel B: National Stock Market Data (DS2)</u>											
H	0.1342 (0.0282)	0.0629 (0.0241)				0.1715 (0.0723)	0.8035 (0.0513)	0.1246 (0.0301)	0.9570 (0.0138)	0.0312 (0.0099)	$\rho_{HS} = 0.4829$ (0.0772)
S	0.0312 (0.0168)	0.1641 (0.0261)				0.0874 (0.0267)	0.7273 (0.0605)	0.1872 (0.0471)			
<u>Panel C: Hang Seng Sectoral Indices Data (DS3)</u>											
F	0.1401 (0.0321)	0.0744 (0.0164)	0.0552 (0.0156)			0.1128 (0.0388)	0.8617 (0.0300)	0.0993 (0.0215)	0.9743 (0.0069)	0.0132 (0.0033)	$\rho_{FP} = 0.8256$ (0.0236)
P	0.0885 (0.0336)	0.1338 (0.0165)	0.0407 (0.0147)	-0.0275 (0.0137)	-0.0202 (0.0122)	0.0982 (0.0306)	0.8822 (0.0209)	0.0908 (0.0155)			$\rho_{FU} = 0.7453$ (0.0283)
U	0.0617 (0.0290)	0.0732 (0.0171)		-0.0394 (0.0150)		0.1508 (0.0393)	0.8209 (0.0284)	0.1204 (0.0198)			$\rho_{PU} = 0.7884$ (0.0258)

Notes: The parameter estimates are the MLE assuming normality. The figures in parentheses are the standard errors. They are calculated using the robustified quasi-MLE (QMLE) covariance matrix of the parameters.

Table 3.3: Estimation Results of the Conditional-Mean Equations of the BEKK(1, 1) Models

Variable	μ	ϕ_1	ϕ_2	ϕ_3	ϕ_4
<u>Panel A: Forex Market Data (DS1)</u>					
D	-0.0017 (0.0128)	0.0472 (0.0207)			
J	-0.0052 (0.0134)	0.0524 (0.0215)			
<u>Panel B: National Stock Market Data (DS2)</u>					
H	0.1188 (0.0299)	0.0574 (0.0269)			
S	0.0270 (0.0186)	0.1574 (0.0262)			
<u>Panel C: Hang Seng Sectoral Indices Data (DS3)</u>					
F	0.1354 (0.0305)	0.0720 (0.0184)	0.0507 (0.0159)		
P	0.0804 (0.0325)	0.1305 (0.0169)	0.0402 (0.0151)	-0.0279 (0.0137)	-0.0214 (0.0133)
U	0.0701 (0.0322)	0.0723 (0.0263)		-0.0483 (0.0172)	

Notes: The parameter estimates are the MLE assuming normality. The figures in parentheses are the standard errors. They are calculated using the robustified quasi-MLE (QMLE) covariance matrix of the parameters.

Table 3.4: Estimation Results of the Conditional-Variance Equations of the BEKK(1, 1) Models

c_{11}	c_{12}	c_{13}	c_{22}	c_{23}	c_{33}	g_{11}	g_{22}	g_{33}	a_{11}	a_{22}	a_{33}
<u>Panel A: Forex Market Data (DS1)</u>											
0.0618 (0.0138)	0.0403 (0.0094)		0.0863 (0.0228)			0.9715 (0.0053)	0.9728 (0.0093)		0.2103 (0.0178)	0.1973 (0.0307)	
<u>Panel B: National Stock Market Data (DS2)</u>											
0.3488 (0.0699)	0.0988 (0.0286)		0.2589 (0.0428)			0.9195 (0.0187)	0.8789 (0.0303)		0.3215 (0.0338)	0.4093 (0.0531)	
<u>Panel C: Hang Seng Sectoral Indices Data (DS3)</u>											
0.1316 (0.0250)	-0.0938 (0.0313)	-0.1467 (0.0458)	-0.1741 (0.0358)	-0.1812 (0.0486)	-0.2528 (0.1525)	0.9654 (0.0060)	0.9552 (0.0109)	0.9548 (0.0382)	0.2309 (0.0183)	0.2709 (0.0297)	0.2561 (0.0876)

Notes: The parameter estimates are the MLE assuming normality. The figures in parentheses are the standard errors. They are calculated using the robustified quasi-MLE (QMLE) covariance matrix of the parameters.

Table 4: Diagnostic Checks for Various Models

Tests	Forex Market	National Stock Markets	Hang Seng Sectoral Indices
	DS1: D-J	DS2: H-S	DS3: F-P-U
<u>Panel A: CC-MGARCH(1, 1) Model</u>			
<i>LF</i>	-1878.23	-4128.73	-7374.26
<i>LMC</i>	4.9138*	10.3442*	8.3589*
$W_{11}(4)$	4.7921	3.9518	6.4373
$W_{22}(4)$	3.8319	1.0694	4.9579
$W_{33}(4)$			5.8914
$W_{12}(4)$	7.6383	5.2031	5.0129
$W_{13}(4)$			7.5183
$W_{23}(4)$			7.6974
<u>Panel B: VC-MGARCH(1, 1) Model</u>			
<i>LF</i>	-1855.80	-4088.90	-7293.40
<i>LR</i>	44.8504*	79.6608*	161.7632*
$W_{11}(4)$	4.7263	3.8607	7.0386
$W_{22}(4)$	3.8157	1.0672	5.0164
$W_{33}(4)$			5.9008
$W_{12}(4)$	6.9215	2.0592	5.2156
$W_{13}(4)$			7.7455
$W_{23}(4)$			6.9466
<u>Panel C: BEKK(1, 1) Model</u>			
<i>LF</i>	-1851.15	-4104.45	-7324.24
$W_{11}(4)$	4.4679	4.0134	12.2758*
$W_{22}(4)$	3.8566	0.7477	6.6085
$W_{33}(4)$			7.8584
$W_{12}(4)$	7.1350	5.5407	0.5775
$W_{13}(4)$			1.2633
$W_{23}(4)$			1.2316

Notes: $W_{ij}(4)$ is Wooldridge's (1991) regression-based diagnostic statistic computed from the standardized residuals of variables i and j based on indicator variables up to lag 4. The indices are according to the order of the coded variables. Thus, $W_{13}(4)$ in the system (F-P-U) is $W_{FU}(4)$. LF is the maximized log-likelihood value. LMC is the Lagrange multiplier test statistic for constant correlation due to Tse (2000). It is approximately distributed as χ_1^2 for a bivariate system and χ_3^2 for the trivariate system when the correlations are time-invariant. LR is the likelihood ratio statistic for $H_0 : \theta_1 = \theta_2 = 0$ in the VC-MGARCH(1, 1) model. Asterisks denote significance at the 5 percent level.

Table 5: Summary Statistics of the Standardized Residuals of Various Models

Variable (Code)	Mean	Std Dev	Minimum	Maximum	Std Skewness	Std Kurtosis	$Q_1(20)$	$Q_2(20)$	No of Obs
<u>Panel A: CC-MGARCH(1, 1) Model</u>									
<u>Forex Market Data (DS1)</u>									
Deutschmark (D)	-0.0001	0.9995	-4.9758	4.1334	-1.3760	10.3514	16.8359	14.7136	2131
Japanese Yen (J)	0.0004	0.9984	-5.8404	4.1482	-11.0287	27.8389	19.1773	11.6364	2131
<u>National Stock Market Data (DS2)</u>									
Hong Kong (H)	-0.0397	0.9991	-8.2506	4.8277	-9.1688	44.4917	22.9629	7.0862	1942
Singapore (S)	-0.0343	1.0000	-6.3858	5.6050	-0.4392	33.4205	31.5455	11.1104	1942
<u>Hang Seng Sectoral Indices Data (DS3)</u>									
Finance (F)	-0.0047	0.9993	-6.2435	4.3125	-2.2710	20.4027	18.5158	21.3844	2440
Properties (P)	-0.0026	0.9992	-6.8961	4.2048	-3.4370	22.2583	36.4885	18.2433	2440
Utilities (U)	0.0054	0.9995	-7.2480	4.4854	-3.9386	27.8046	25.4859	10.8179	2440
<u>Panel B: VC-MGARCH(1, 1) Model</u>									
<u>Forex Market Data (DS1)</u>									
Deutschmark (D)	-0.0019	0.9976	-5.1997	4.0989	-1.5961	10.6562	16.8138	13.6033	2131
Japanese Yen (J)	-0.0063	1.0015	-5.8539	4.1495	-11.0059	27.8081	19.1706	11.6673	2131
<u>National Stock Market Data (DS2)</u>									
Hong Kong (H)	-0.0482	1.0019	-8.4556	4.5876	-9.7967	46.5185	25.6588	7.3562	1942
Singapore (S)	-0.0410	1.0086	-6.4275	5.6201	-0.3672	33.1772	32.0569	11.1440	1942
<u>Hang Seng Sectoral Indices Data (DS3)</u>									
Finance (F)	-0.0271	1.0012	-6.4599	4.3445	-2.1722	20.8953	17.6054	25.0739	2440
Properties (P)	-0.0207	1.0034	-7.2261	4.0554	-3.6343	23.2231	36.6489	13.4232	2440
Utilities (U)	-0.0155	0.9975	-7.5899	4.3811	-4.2861	29.6450	25.2002	9.0371	2440

Table 5: ..Continued

Variable (Code)	Mean	Std Dev	Minimum	Maximum	Std Skewness	Std Kurtosis	$Q_1(20)$	$Q_2(20)$	No of Obs
<u>Panel C: BEKK(1, 1) Model</u>									
<u>Forex Market Data (DS1)</u>									
Deutschmark (D)	0.0030	0.9932	-5.0577	4.0577	-1.5112	10.5148	16.6838	15.1533	2131
Japanese Yen (J)	-0.0007	0.9954	-5.9277	4.1335	-11.5370	28.4453	19.5022	12.5101	2131
<u>National Stock Market Data (DS2)</u>									
Hong Kong (H)	-0.0359	0.9874	-8.4438	4.3589	-10.3784	48.5443	26.2642	9.8984	1942
Singapore (S)	-0.0357	0.9996	-6.5838	5.5635	-0.8210	33.5270	33.7262	10.5492	1942
<u>Hang Seng Sectoral Indices Data (DS3)</u>									
Finance (F)	-0.0224	0.9822	-6.6095	4.0304	-3.2424	25.1857	20.2667	64.4039	2440
Properties (P)	-0.0143	0.9849	-7.2597	4.0862	-3.9546	24.4061	36.8795	16.9989	2440
Utilities (U)	-0.0159	0.9792	-7.6392	4.5386	-3.5271	33.4623	25.2280	25.3953	2440

— Properties (P)
- - Utilities (U)



Figure 4: Conditional Correlation Coefficients of [D, J]

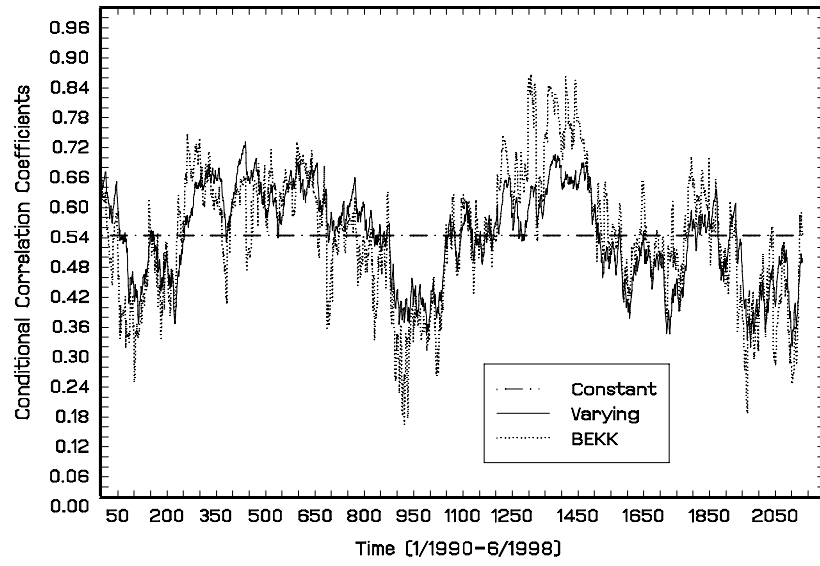


Figure 5: Conditional Correlation Coefficients of [H, S]

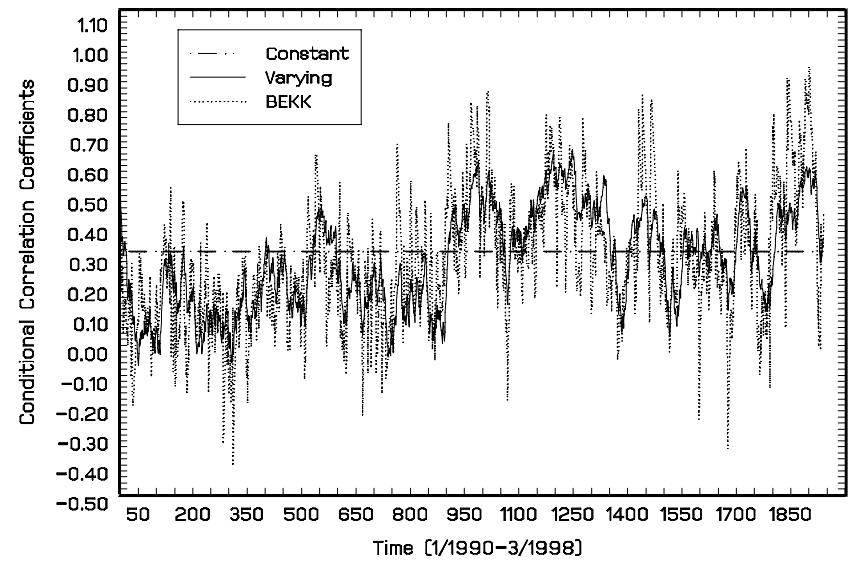


Figure 6: Conditional Correlation Coefficients of [F, P]

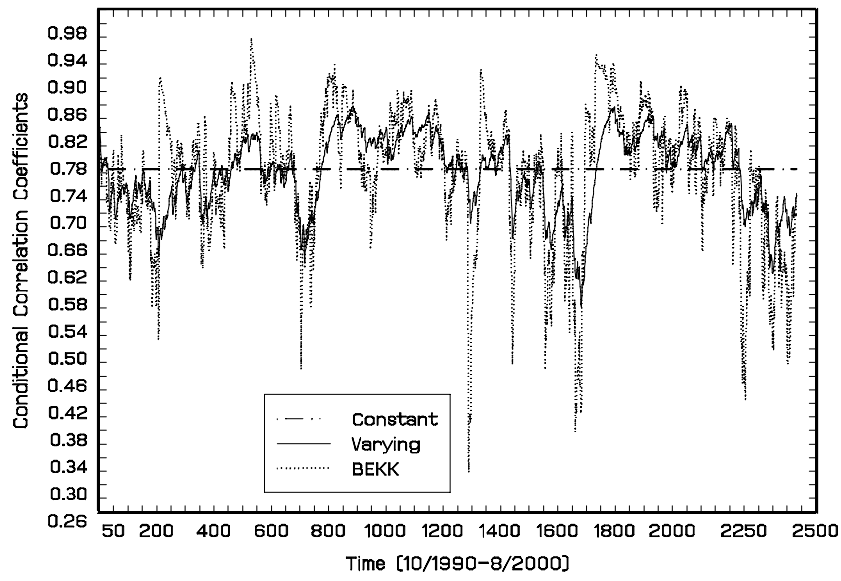


Figure 7: Conditional Correlation Coefficients of [F, U]

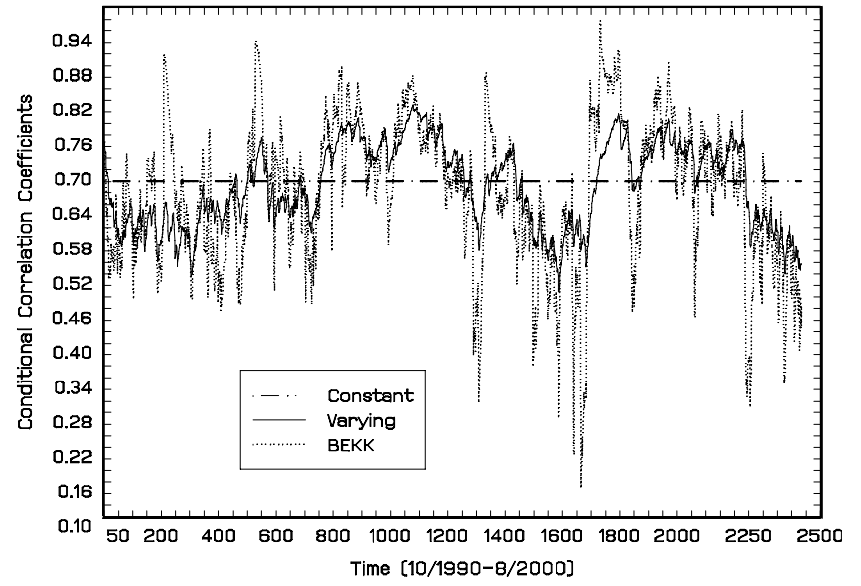


Figure 8: Conditional Correlation Coefficients of [P, U]

