Evaluating the Hedging Performance of the Constant-Correlation GARCH Model[*]

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Abstract: This paper compares the performances of the hedge ratios estimated from the OLS (ordinary least squares) method and the constant-correlation VGARCH (vector generalized autoregressive conditional heteroscedasticity) model. These methods are evaluated based on the out-of-sample optimal hedge ratio forecasts. A systematic comparison is provided by examining ten spot and futures markets covering currency futures, commodity futures and stock index futures. Using a recently proposed test (Tse (2000)) for the constant-correlation assumption, we find that the assumption cannot be rejected for eight of the ten series. To gain the maximum benefit of a time-varying hedging strategy we keep the estimation data up-to-date for the re-estimation of the hedge ratios. Both the constant hedge ratio (using OLS) and the time-varying hedge ratio (using constant-correlation VGARCH) are re-estimated on a day-by-day rollover, and the post-sample variances of the hedged portfolios are examined. We find that the OLS hedge ratio performs better than the VGARCH hedge ratio. This result may be another indication that the forecasts generated by the VGARCH models are too variable.

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1. Introduction

Traditional approach to commodity futures hedging adopts a one to one ratio. That is, for any given spot position, an equal amount of futures position should be undertaken to reduce the risk. The portfolio approach recognizes the existence of basis risk and determines the optimal futures position by minimizing the variance of the spot-futures portfolio. The optimal hedge ratio is then equal to the covariance between the spot and futures returns divided by the variance of the futures return. Suppose we construct a linear regression model with the spot return being the dependent variable and the futures return being the independent variable. The OLS (ordinary least squares) estimate of the slope is the estimated optimal hedge ratio.

The conventional OLS approach assumes that the second moments are constant over time. It is well known in the finance literature that asset returns typically exhibit time-varying conditional heteroscedasticity. Thus, to enhance the estimation results, it is important to take account of the possible time-varying nature of the second moments. The GARCH (generalized autoregressive conditional heteroscedasticity) models proposed by Engle (1982) and Bollerslev (1987) are particularly useful for this purpose and have since been extensively applied in the futures market literature.

For the purpose of estimating the optimal hedge ratio, the conditional covariance between the spot and futures returns is required under the time-varying framework. Thus, a bivariate conditional heteroscedasticity model has to be constructed. To this effect, there are several variants of VGARCH (vector GARCH) models that can be applied. Bollerslev, Engle and Wooldridge (1988) extended the GARCH representation in the univariate case to the vectorized conditional variance matrix, resulting in the

VECH representation model. One disadvantage of the VECH model, however, is that it cannot ensure the conditional variance-covariance matrix to be positive semidefinite. Lien and Luo (1994), for example, found that it is not unusual for the conditional variance-covariance matrix of the models to violate the positive semidefinite condition. Engle and Kroner (1995) proposed the BEKK (named after Baba, Engle, Kraft and Kroner) specification. This specification is fairly general and, unlike the VECH model, can ensure the conditional variance-covariance matrix to be positive semidefinite.

Baillie and Myers (1991) compared the hedge ratios estimated from the OLS and VGARCH models. They considered two versions of VGARCH models, namely, the diagonal VECH form and the BEKK form. Using six commodity prices, they found that the time-varying hedge ratios computed from the VGARCH models exhibit significant variations. Indeed, the series of estimated hedge ratios are found to be nonstationary. Their analysis of the hedged portfolios based on the OLS and the time-varying hedge ratios indicate that the latter performs better in its ability to reduce risk.

Baillie and Myers' out-of-sample comparison was, however, based on contracts that were quite separated in time (1986 versus 1982). Also, a somewhat peculiar comparison was made when the estimated model based on the data in 1986 was used to calculate the hedge ratio for the data in 1982. As such, it is difficult to judge whether their conclusions are representative of the typical hedging performance. In this paper, we report a systematic comparison of the empirical performance of the OLS and VGARCH hedge ratios. We re-estimate the optimal hedge ratios of both strategies on a day-by-day rollover, each time keeping the number of estimation observations unchanged. Assuming the forecast horizon to be one day, we compare the hedging performance of the strategies based on the most currently updated hedge ratios. By comparing the hedging performance in this way, we alleviate the problem of structural changes. Also, the maximum benefit of a time-varying hedging strategy can only be obtained when the estimation data are kept up-to-date. As the results are based on one-period post-sample hedge, the problem of in-sample data smoothing does not apply. To the best of our knowledge, no study in the literature has made use of day-by-day updating of the conditional heteroscedasticity models in making comparison between the time-varying and time-invariant models. In our view, updating is important in providing a fair comparison between the two types of models.

As the VGARCH models have to be re-estimated daily in the post-sample period, it is important to choose a model that is computationally convenient. To this effect, we adopt the constant-correlation GARCH (CC-GARCH) model proposed by Bollerslev (1990). This model is computationally simple and is relatively easy to ensure the positive semidefiniteness of the conditional variance-covariance matrix during the optimization. Empirical research that uses this method includes: Bollerslev (1990), Kroner and Claessens (1991), Kroner and Sultan (1991), Kroner and Sultan (1993), Park and Switzer (1995) and Lien and Tse (1998).

We examine ten pairs of spot and futures series covering currency futures, commodity futures and stock index futures. For each contract the complete data set is divided into two sample periods. One sample is used for the base estimation and diagnostics, while the other sample is used for re-estimation and comparison of the hedge performance. Model diagnostics show that the CC-GARCH model is generally acceptable for the data, although two of the ten bivariate series show evidence of violation in the constant-correlation assumption. In the post-sample analysis both the constant hedge ratio (using OLS) and the time-varying hedge ratio (using CC-GARCH) are re-estimated on a day-by-day rollover by simultaneously augmenting the next observation and dropping the first observation. The mean and the variance of the hedged portfolio returns in the one-day post-sample hedges summarize the hedging performance of the two hedge ratios. In each pair of markets, we find that the OLS hedge ratio performs better than the CC-GARCH hedge ratio. Although the magnitude of the difference may not be large for some markets, the results clearly indicate that the CC-GARCH model, while requiring additional computational costs, fails to improve the futures hedging.

The plan of this paper is as follows. In Section 2 we describe the methods of estimating the optimal hedge ratio. The data used in this study are described in Section 3. Section 4 discusses the performances of the time-varying and OLS strategies. Some concluding remarks are given in Section 5.

2. Estimation of Optimal Hedge Ratio

We consider a one-period model. At the beginning of the period an economic agent has a given non-tradable spot position on a specific security. To reduce the risk exposure, the agent goes short in the futures market. The futures position is chosen to minimize the variance of the hedged portfolio. We define the optimal hedge ratio as the amount of futures position per unit spot position such that the hedged portfolio variance is minimized. Denoting r_s and r_f as the returns on the spot and futures, respectively, the optimal hedge ratio, denoted by h, is given by $h = \mathrm{Cov}(r_s, r_f)/\mathrm{Var}(r_f)$.

To estimate the hedge ratio, a conventional method involves estimating the following linear regression model:

$$r_{st} = \alpha + \beta r_{ft} + \varepsilon_t,$$

where r_{st} and r_{ft} are the spot and futures returns, respectively, for period t, and ε_t is the disturbance term. The OLS estimator of β provides an estimate for the optimal hedge ratio h.

The OLS method assumes that the second moments remain unchanged over time. This assumption, however, is not supported by most empirical research. To account for the possibility of time-varying moments, the hedge ratio may be estimated from the conditional moments directly. To this effect, the VGARCH models may be adopted. Of the various versions of VGARCH models available in the literature, the VECH model provides one of the most flexible functional forms. A disadvantage of the VECH model, however, is that it does not ensure the conditional variance-covariance matrix of the spot and futures returns to be positive semidefinite. As such, it is not suitable for extensive computation when the model has to be re-estimated for post sample comparison. An alternative is the BEKK specification suggested by Kroner and Engle (1995). Bera, Garcia and Roh (1997), however, showed that this specification produced the worst hedging performance when compared to the OLS and random-coefficient hedge ratios.

In this paper, we consider the constant-correlation GARCH (CC-GARCH) model. For parameter parsimony, we consider a CC-GARCH(1, 1) model. The conditional mean of the returns are postulated to follow AR processes. Thus, the conditional-mean and conditional-variance-covariance equations are assumed to be given by:

$$r_{st} = \theta_{s0} + \sum_{j=1}^{J} \theta_{sj} r_{s,t-j} + \varepsilon_{st},$$

$$r_{ft} = \theta_{f0} + \sum_{k=1}^{K} \theta_{fk} r_{f,t-k} + \varepsilon_{ft},$$

$$\sigma_{st}^2 = \gamma_s + \alpha_s \sigma_{s,t-1}^2 + \beta_s \varepsilon_{s,t-1}^2,$$

$$\sigma_{ft}^2 = \gamma_f + \alpha_f \sigma_{f,t-1}^2 + \beta_f \varepsilon_{f,t-1}^2,$$

$$\sigma_{sft} = \rho \sigma_{st} \sigma_{ft},$$

where γ, α and β are all positive, with $\alpha_i + \beta_i \leq 1$ for $i = s, f$. Thus, while the variance and covariances are time varying, the correlation coefficient is assumed to be constant.

Bollerslev (1990) pointed out that under the assumption of constant correlation, the maximum likelihood estimate (MLE) of the correlation matrix is equal to the sample correlation matrix. As the sample correlation matrix is positive semidefinite, the positive semidefiniteness of the conditional variance-covariance matrix can be ensured when the conditional variances are all positive. In addition, when the correlation matrix is concentrated out of the likelihood function further simplification is achieved in the optimization. In this paper all calculations done were coded in GAUSS with the module MAXLIK, and performed on PCs with 200-MH Pentium processors.

The CC-GARCH model was applied by Kroner and Sultan (1993) to obtain the optimal hedge ratios of currency futures and by Park and Switzer (1995) to obtain the optimum hedge ratios of stock index futures. While Baillie and Myers (1991) provided some results on the hedging performances of the VECH-GARCH and BEKK models, there has been no systematic study of the hedging effectiveness using the CC-GARCH model. We report that we attempted to do a comparison that incorporates the BEKK model. However, we experienced problems in getting convergence in the estimation of the BEKK model while trying to complete the post-sample re-estimation. Despite efforts to cut down on the number of post-sample re-estimations through sub-sampling in the post-sample period, convergence was not achieved. Although some results were obtained for a limited number of contracts, a meaningful comparison of other contracts was not available. Thus, the attempt to incorporate the BEKK model was aborted.

3. Data Description

The data used in this paper were obtained from the Commodity Systems Inc. Ten contracts were used (for convenience we call the assets underlying the contracts commodities). These consist of three currency futures contracts (British Pound (BP), Deustchmark (DM) and Japanese Yen (JY)), five commodity futures contracts (soy bean oil (BO), wheat (KW), crude oil (CL), corn (C) and cotton (CT)) and two stock index futures contracts (NYSE composite (YX) and S&P 500 (SP)), making up a total of ten contracts. Daily spot and futures prices were extracted for the period January 1988 through June 1998. The near contract was used, with rollover occurring about one week before maturity in most cases. The trading volume was used as a criterion in deciding the actual rollover date. Daily return was then calculated as the differenced logarithmic prices (in percentage). Care has been taken to ensure that at the rollover the difference is taken over the same contract so that the return of the futures is appropriately calculated.

The total number of observations for each commodity varies from 2600 to 2658. For each commodity the first 1500 observations were used to estimate the basic CC-GARCH models and the diagnostics of the models were calculated. This part of the sample is termed Period I. The remaining sample period is called Period II.

We calculated some summary statistics for the various series. To conserve space, these results are not reproduced here (see Lien, Tse and Tsui (1999) for the details). The following points, however, were observed. First, the Box-Pierce portmanteau statistics indicated significant serial correlation in the squared returns, which suggested the need to model the conditional heteroscedasticity. Second, there was evidence of serial correlation in some of the return series. Thus, a conditional-mean equation to take account of the serial correlation in the returns is required. Third, the spot and futures returns had similar distribution statistics except for the skewness and the kurtosis in some cases. Fourth, the summary statistics showed signs of variations across the two different subperiods, suggesting that there might be a need to incorporate time-varying parameters. We take account of this last point by updating the estimated parameters as time moves on.

To lessen the computations of the estimation for the conditional-variance-covariance equations, we perform the estimation in two stages. In the first stage, we estimated the mean equations of the returns on the spot and futures using univariate autoregressive (AR) filters estimated from OLS. In the second stage we fitted bivariate CC-GARCH(1, 1) models to the filtered spot and futures returns series. As the estimated parameters of the conditional mean and variance are asymptotically uncorrelated, this two-stage method is justified.

Table 1 summarizes the results of the AR filters. DM is the only case for which the pure random walk is adequate for both the spot and futures. For BP, the pure random walk is adequate for the futures, but not for the spot. Overall, the AR filters for the spot and futures series are quite similar. Some results for the unit root tests for the spot and futures prices as well as returns are also given. The augmented Dickey-Fuller (ADF) test is used to test for nonstationarity. As expected, all price series are nonstationary, while all return series are stationary.

4. Performance of CC-GARCH Hedging

The estimation results for the CC-GARCH models using the data from Period I are presented in Table 2. To examine the adequacy of the models, we report a sequence of Q statistics. To test for the constant-correlation assumption, we calculate the LMC statistic due to Tse (2000). LMC is asymptotically distributed as a χ^2 with 1 degree of freedom under the null of constant correlation. It can be seen that the constant-correlation assumption is acceptable for all markets except for JY and CT. As expected, the correlations between the spot and futures returns are very high. The commodity markets display the smallest correlation, ranging from 0.5294 (wheat) to 0.9437 (soy bean oil). The correlation is at its highest, about 0.97, for the currency markets. Overall, the diagnostics show that the CC-GARCH(1, 1) model is satisfactory for the spot and futures data, with the possible exception of JY and CT.

From Table 2 it can be seen that the sums of the estimates of α and β are rather large, indicating that both spot and futures returns exhibit strong persistence in volatility. The commodity markets exhibit the strongest persistence, followed by the currency markets. The stock indices demonstrate the smallest (yet highly significant) persistent effects. Also, with the exception of the stock index markets, the estimated GARCH equations for the spot and futures returns are very similar. The returns on the stock index futures exhibit much stronger persistence than the returns on the corresponding spot itself.

Denoting T as the estimation sample size, we estimate the optimal one-day post-estimation hedge ratio by $h_T = \sigma_{sf,T+1}/\sigma_{ff,T+1}$. Based on h_T the return of the hedged portfolio at time $T + 1$ is calculated. Thereafter we incorporate the observation for time $T + 1$ and the observation for time 1 is deleted, keeping the size of the estimation sample fixed. The AR filters are re-estimated, with the orders of the filters kept unchanged. We then re-estimate the CC-GARCH(1, 1) model and construct the optimal hedge ratio subsequently. We continue this rollover estimation method for all post-sample data points. As a result, for each market we obtain a series of hedged portfolio returns generated by the GARCH strategy. Similarly, the rollover method was applied to the OLS hedge ratio to obtain a series of hedged portfolio returns for each market. Comparisons of the portfolio returns determine the performance of the GARCH strategy relative to the OLS strategy.

Table 3 documents the results of the comparison. On average, the OLS hedge ratio is larger than the GARCH hedge ratio for the currency markets. For the commodity and stock indices, the relative sizes of the hedge ratios vary. As expected, the variations of the OLS hedge ratios are small. Figures 1 to 10 display the OLS and GARCH hedge ratios for each pair of markets over the post-sample period. Soy bean oil presents an interesting case in which the variations of the GARCH hedge ratios mainly occur in the early period (see Figure 4). In the later period, the hedge ratios are rather stable. The ADF tests reported in Panel B of Table 3 indicate that the null hypothesis that the optimal GARCH hedge ratios have a unit root is rejected for all markets. This result is in contrast with those of Baillie and Myers (1991), who reported that the (in-sample) optimal GARCH hedge ratios are nonstationary.

It is interesting to note that while the GARCH hedge ratio (in comparison to the OLS ratio) varies more in some markets than in others, a uniform conclusion is obtained concerning the hedging performance. In each market, we find that the OLS hedge ratios provide smaller hedged portfolio variance than the GARCH hedge ratios. The difference is generally very small with the exception of crude oil (and perhaps soy bean oil). In the crude oil market, the GARCH hedge ratio incurs near 20% additional risk as compared to the OLS hedge ratio. The incremental risk is around 5% for the soy bean oil market. We conclude that, while incurring additional computational costs, the CC-GARCH hedge strategy provides no benefits in risk reduction beyond the OLS method.

5. Conclusions

Conditional heteroscedasticity has been well accepted as one of the main characteristics of financial time series. There is, however, to date no definite conclusions concerning the usefulness of the GARCH models in improving hedging performance. In this paper, we adopt the constant-correlation GARCH model for ten selected markets. We employ rollover methods to construct the hedged portfolios so that only one-step-ahead forecasts of the second moments are required. In each market the post-sample comparisons show that the GARCH strategy cannot outperform the ordinary least squares (OLS) hedge strategy. In one case, the GARCH strategy incurs 20% more risk than the OLS strategy. Given the high computational costs for the GARCH estimation, our results indicate that the GARCH models should not be considered for hedging purposes though they may remain useful for data description.

It is well known that the GARCH models often produce variance forecasts that are too variable. Fitted GARCH models often exhibit high volatility persistence. Recently, some authors have suggested models that attempt to relieve the volatility persistence. The regime-switching models proposed by Hamilton and Susmel (1994) and Klaassen (1998) appear to be successful in providing better variance forecasts. In view of the results in this paper, it would be interesting to see if the regime-switching models can improve the hedging performance of futures over the traditional GARCH models.

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Table 1: Unit-Root Tests and Univariate Autoregressive Filters

Commodity (Code)	ADF Test				Order of Autoregressive Filters for Return Series	
	Price Series		Return Series			
	p_f	p_s	r_f	r_s	r_f	r_s
British Pound (BP)	-1.856	-1.978	-9.908	-9.872	-	1
Deutschmark (DM)	-1.976	-2.049	-12.235	-10.241	-	-
Japanese Yen (JY)	-0.514	-0.528	-10.154	-10.255	6, 10	6, 10
Soy Bean Oil (BO)	-2.267	-2.271	-10.770	-10.755	1, 2	1, 2
Wheat (KW)	-1.186	-1.755	-9.636	-10.553	1, 8	1, 8
Crude Oil (CL)	-2.186	-2.224	-11.025	-10.884	3	3, 5
Corn (C)	-0.305	0.263	-9.207	-9.366	1, 2, 7	2, 7
Cotton (CT)	-1.697	-1.786	-10.722	-11.353	2	2
NYSE Composite (YX)	-0.936	-0.993	-11.031	-10.707	1, 7	1, 7
S&P 500 (SP)	-1.004	-1.002	-11.030	-10.915	7	7

Notes: p_f is the price of the futures and p_s is the price of the spot. Similarly, r_f is the return of the futures and r_s is the return of the spot. ADF Test is the augmented Dickey-Fuller statistic for the hypothesis that the price/return series have a unit root. The (lower tail) critical values are -3.49 and -2.89 at the 1% and 5% levels, respectively. The univariate autoregressive filters are estimated using OLS.

Table 2: Estimated Constant-Correlation VGARCH(1, 1) Models

Parameters	Commodity									
	BP	DM	JY	BO	KW	CL	C	CT	YX	SP
γ_f	0.1113	0.0268	0.0832	0.1237	0.0635	0.1526	0.0761	0.1071	0.1797	0.1787
	(0.0624)	(0.0081)	(0.0426)	(0.0437)	(0.0216)	(0.0834)	(0.0257)	(0.0650)	(0.0875)	(0.0633)
α_f	0.7555	0.8810	0.7110	0.8170	0.8302	0.7856	0.8438	0.8857	0.7003	0.7088
	(0.1119)	(0.0205)	(0.1141)	(0.0337)	(0.0413)	(0.0686)	(0.0359)	(0.0484)	(0.0902)	(0.0703)
β_f	0.0645	0.0739	0.1126	0.1248	0.1141	0.1992	0.0974	0.0508	0.0755	0.0680
	(0.0220)	(0.0146)	(0.0323)	(0.0258)	(0.0293)	(0.0665)	(0.0208)	(0.0143)	(0.0291)	(0.0233)
γ_s	0.1395	0.0255	0.0652	0.1068	0.0725	0.1133	0.0765	0.1517	0.2002	0.2752
	(0.0740)	(0.0077)	(0.0382)	(0.0408)	(0.0259)	(0.0838)	(0.0256)	(0.0727)	(0.0861)	(0.1004)
α_s	0.6765	0.8759	0.7594	0.8423	0.8380	0.8195	0.8638	0.8785	0.5433	0.5056
	(0.1321)	(0.0196)	(0.1073)	(0.0356)	(0.0352)	(0.0759)	(0.0293)	(0.0407)	(0.1371)	(0.1318)
β_s	0.0862	0.0809	0.0999	0.1129	0.1129	0.1750	0.0919	0.0523	0.1116	0.1033
	(0.0218)	(0.0147)	(0.0322)	(0.0304)	(0.0263)	(0.0747)	(0.0183)	(0.0138)	(0.0202)	(0.0179)
ρ	0.9743	0.9729	0.9700	0.9437	0.5294	0.8798	0.8800	0.8012	0.9616	0.9635
	(0.0019)	(0.0020)	(0.0028)	(0.0095)	(0.0220)	(0.0125)	(0.0118)	(0.0170)	(0.0025)	(0.0023)
LMC	2.77	0.17	4.85	2.02	0.58	3.01	0.01	9.07	2.88	1.99
LL	1527.81	1629.38	1839.20	-779.29	-1384.36	-2322.03	-836.80	-1638.69	1071.07	950.66
$Q_f(20)$	23.05	15.32	20.92	14.23	34.03	32.87	21.83	30.48	15.64	14.86
$Q_s(20)$	19.06	11.61	20.36	14.99	25.10	24.45	24.76	35.97	12.00	15.25
$Q_{ff}(20)$	26.63	16.15	21.16	16.32	18.38	18.07	23.14	21.45	3.06	5.46
$Q_{fs}(20)$	28.29	16.29	27.36	17.55	23.58	19.51	23.15	31.68	6.03	9.34
$Q_{ss}(20)$	31.83	15.77	36.52	17.90	15.98	18.24	19.48	29.36	12.24	16.78

Notes: LMC is the Lagrange Multiplier test for the constancy of the correlation coefficient suggested by Tse (1999). The statistic is approximately a χ^2_1 if the correlation coefficient is constant. LL stands for the log-likelihood value. $Q_i(20)$; $i = f, s$; are the Box-Pierce portmanteau statistics of the standardized residuals of the returns of the futures and spot based on autocorrelation coefficients of order up to 20. $Q_{ii}(20)$; $i = f, s$; are the portmanteau statistic of the squares of the standardized residuals of the returns of the futures and spot based on autocorrelation coefficients of order up to 20. Similarly, $Q_{fs}(20)$ is the portmanteau statistic of the cross product of the standardized residuals of the return of the futures and spot. The conditional variances and covariances are given by: $\sigma^2_{it} = \gamma_i + \alpha_i \sigma^2_{i,t-1} + \beta_i \epsilon^2_{i,t-1}$, for $i = f, s$, and $\sigma_{ij,t} = \rho \sigma_{it} \sigma_{jt}$.

Table 3: Statistics of Estimated Hedge Ratios and Comparison of Hedged Portfolios

Statistics of Hedge Ratios and Hedged Portfolios	Commodity									
	BP	DM	JY	BO	KW	CL	C	CT	YX	SP
Panel A: OLS Method										
Mean	0.9498	0.9652	0.9570	0.9985	0.5771	0.9812	0.9928	0.9470	0.8329	0.8996
Std Dev	0.0021	0.0019	0.0063	0.0058	0.0161	0.0117	0.0194	0.0187	0.0184	0.0105
Minimum	0.9458	0.9601	0.9482	0.9862	0.5571	0.9572	0.9646	0.9011	0.7970	0.8841
Maximum	0.9544	0.9695	0.9691	1.0098	0.6277	1.0067	1.0463	0.9766	0.8587	0.9188
Mean of Hedged Portfolio	-0.0028	0.0042	0.0151	-0.0031	-0.0552	-0.0546	-0.0149	-0.0023	0.0201	0.0187
Var of Hedged Portfolios	0.0135	0.0196	0.0274	0.1010	1.3955	1.5592	0.3976	0.4941	0.0391	0.0386
Number of Re-estimations/Portfolios	1140	1140	1140	1100	1158	1131	1158	1138	1145	1144
Panel B: Constant-Correlation VGARCH(1, 1) Method										
Mean	0.9471	0.9605	0.9559	1.0052	0.5088	1.0098	1.0091	0.9449	0.8370	0.8954
Std Dev	0.0244	0.0260	0.0391	0.0414	0.0900	0.2212	0.1367	0.0847	0.0413	0.0366
Minimum	0.8573	0.8428	0.7208	0.8290	0.3036	0.5990	0.5927	0.6456	0.6843	0.7513
Maximum	1.0296	1.0429	1.0940	1.4553	1.0605	3.0076	1.9586	1.6891	0.9949	1.0266
Mean of Hedged Portfolios	-0.0027	0.0039	0.0151	-0.0052	-0.0541	-0.0521	-0.0039	-0.0036	0.0203	0.0192
Var of Hedged Portfolios	0.0136	0.0198	0.0278	0.1066	1.4004	1.8632	0.4123	0.5093	0.0395	0.0387
ADF Test	-5.7608	-5.9513	-7.7705	-6.5135	-7.0460	-6.0589	-4.3885	-6.0409	-4.7753	-5.8963
Number of Re-estimations/Portfolios	1140	1140	1140	1100	1158	1131	1158	1138	1145	1144

Notes: The first four rows (Mean, Std Dev, Minimum and Maximum) in each panel provide the statistics for the estimated one-period hedge ratios. The next two rows (Mean of Hedged Portfolios and Var of Hedged Portfolios) provide the results for the post-sample hedge. The ADF Test in Panel B is the augmentedd Dickey-Fuller statistic for the null hypothesis that the GARCH hedge ratios have a unit root. The (lower tail) critical values are -3.49 and -2.89 at the 1% and 5% levels, respectively.

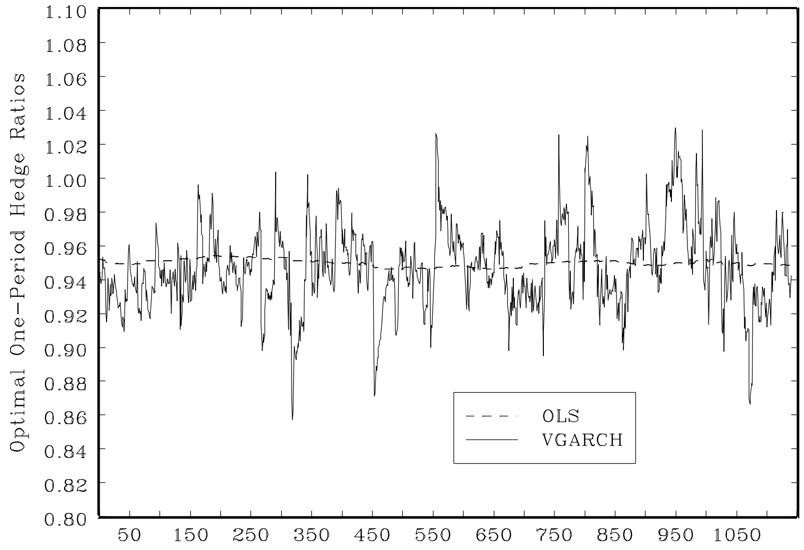

Figure 1: Comparison of Optimal Hedge Ratios for BP

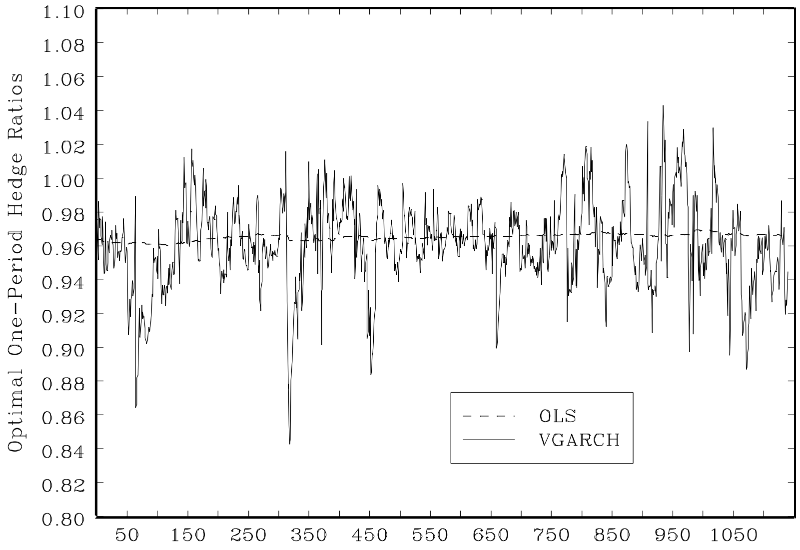

Figure 2: Comparison of Optimal Hedge Ratios for DM

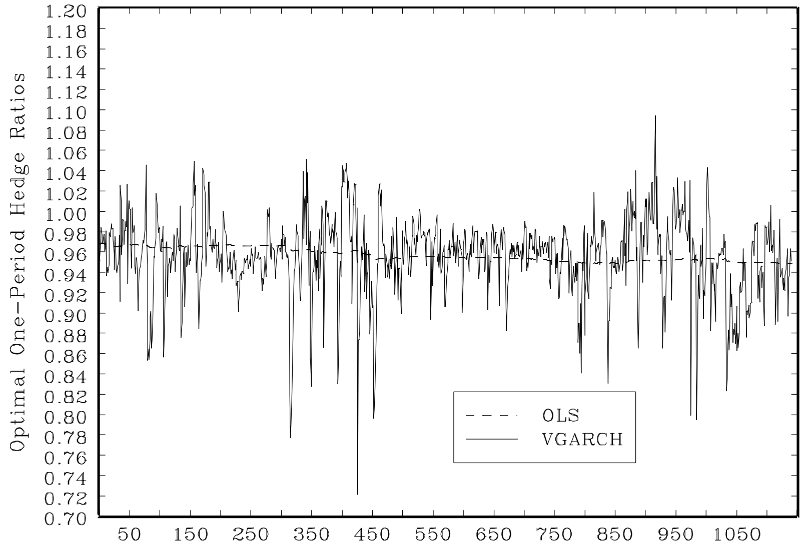

Figure 3: Comparison of Optimal Hedge Ratios for JY

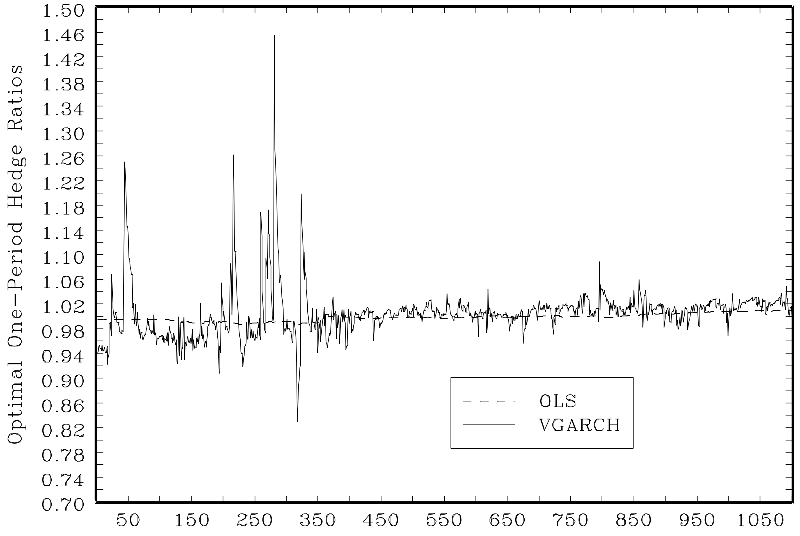

Figure 4: Comparison of Optimal Hedge Ratios for BO

Figure 5: Comparison of Optimal Hedge Ratios for KW

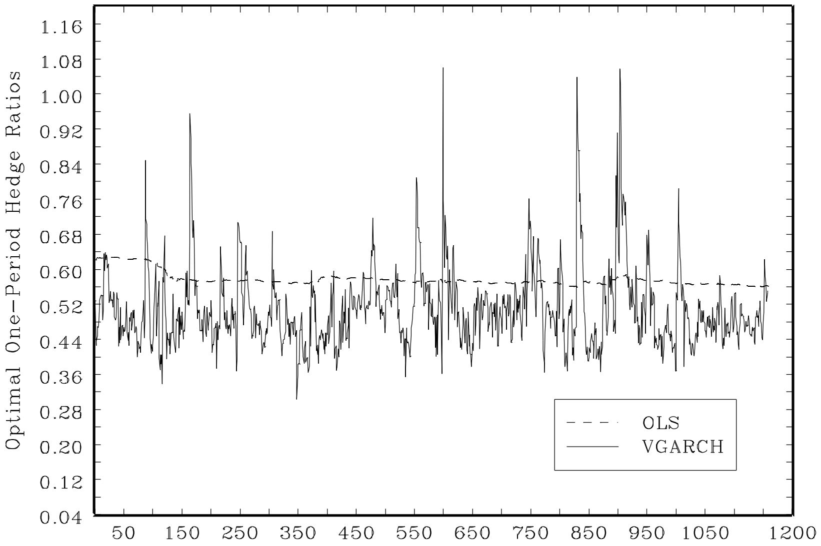

Figure 6: Comparison of Optimal Hedge Ratios for CL

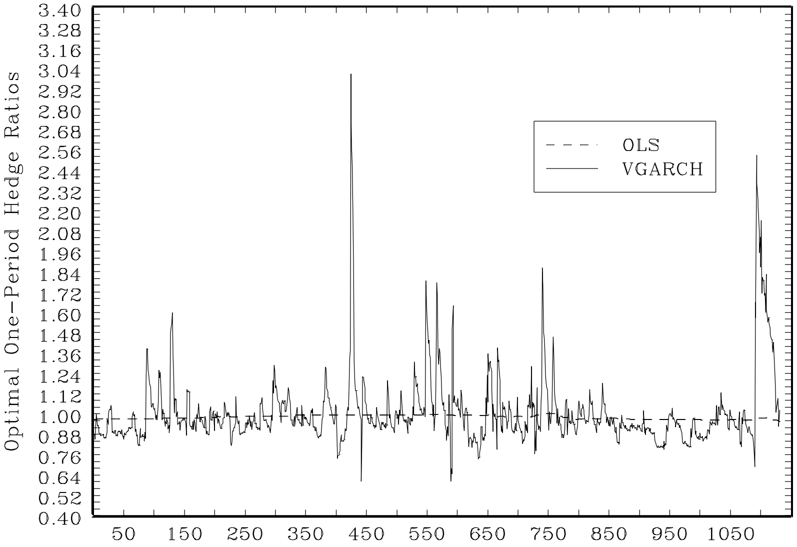

Figure 7: Comparison of Optimal Hedge Ratios for C

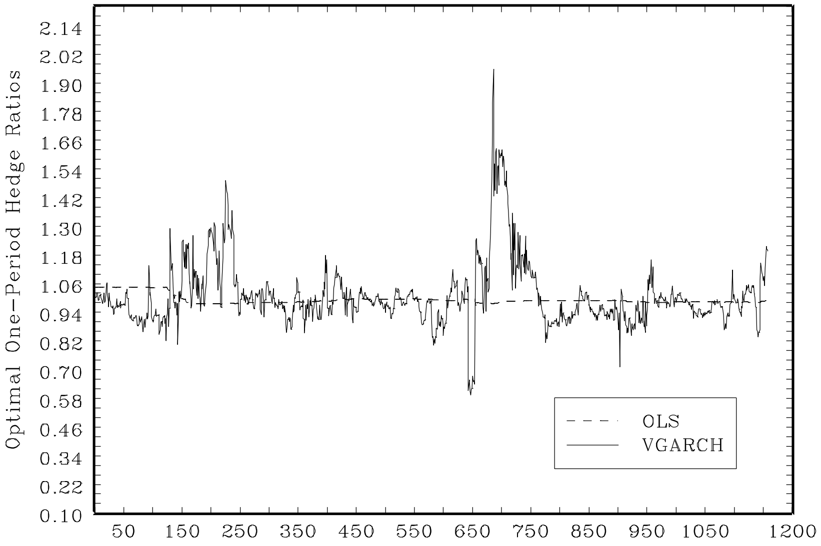

Figure 8: Comparison of Optimal Hedge Ratios for CT

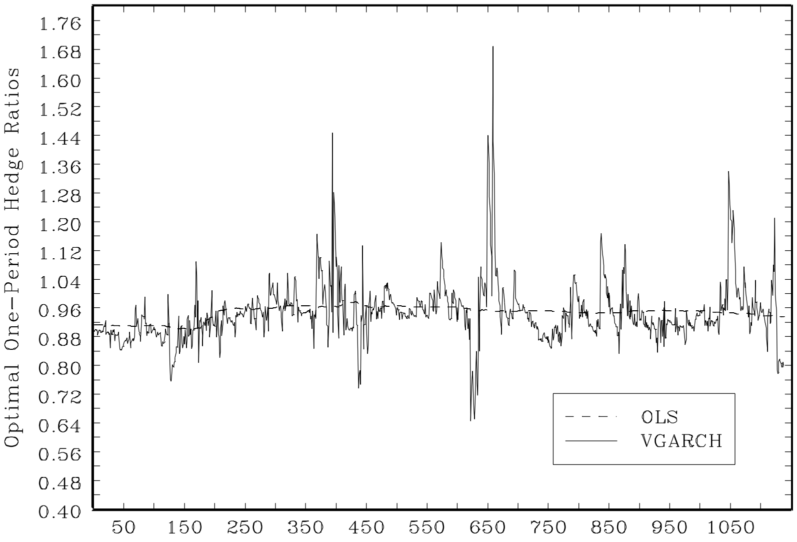

Figure 9: Comparison of Optimal Hedge Ratios for YX

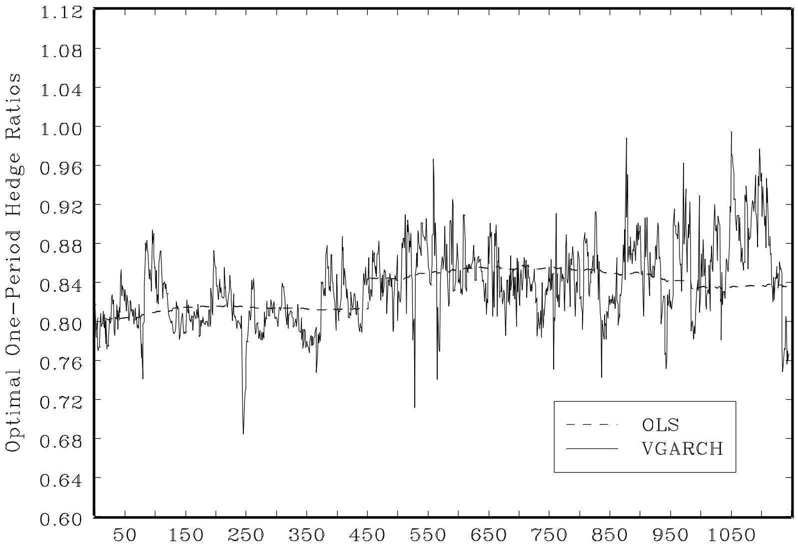

Figure 10: Comparison of Optimal Hedge Ratios for SP